Worcester County Mathematics League

WOCOMAL Varsity Meet #3

Coaches' Booklet

February 4, 2004

Round 1:   Similarity and Pythagorean Theorem

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Determine the area of a circle inscribed in a right triangle whose legs measure 8cm and 15 cm.

2.   Rectangle ABCD has adjacent sides of lengths 7 and 24. What is the shortest distance between vertex A and diagonal BD?

3.   An isosceles right triangle ABC is given with right angle B. Point D is on side BC. Angle bisector AD has length 4. Determine the area of triangle ABC.

ANSWERS

(1 pt.)   1. _____cm^2

(2 pts.)   2. _____

(3 pts.)   3. _____

St. John's, Worcester Academy

Round 2:   Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Determine the greatest common factor of:

$$2x^4 - x^3 - x^2 ; \quad 2x^2 + x - 3 ; \quad \text{and} \quad x^3 - x^2 - x + 1$$

2.   If the mean of two integers is 11 and their product is 117, what is the absolute value of the difference between the two integers?

3.   Jake gave Patti a 5-yard head start in a 100-yard dash and Jake lost by only ¼ yard. In how many **more** yards, beyond 100 yards, would Jake have overtaken Patti?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Notre Dame, Leicester, Assabet Valley

Round 3:   Functions (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   If $f(x) = 2x + 5$ and $g(x) = x - 3$, find the inverse of $f(g(x))$ and write your
         answer as a single fraction in reduced form.

2.   Let $f(x) = 9x + 1$ and $g(x) = x^2$. Find all values of x which satisfy the equation
         $f(g(x)) = g(f(x))$.

3.   If $f(1) = 3$ and $f(x+y) = f(x)f(y)$, determine the value of $f(16)$ and write your
         answer in "prime factorization" form (i.e. $24 = (2^3)(3)$).

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Auburn, Hudson, Worcester Academy

Round 4:   Combinatorics

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1.   How many distinct ways can the letters of the word "**ICICLE**" be arranged?

2.   If $_8C_x = 28$, find all possible values of x.

3.   How many positive 6-digit numbers contain the digit "1" exactly once, the digit "2" exactly twice, and the digit "3" exactly three times? (for example, 122333 or 231323)

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Douglas, Worcester Academy, Mass Academy

Round 5:   Analytic Geometry (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Find the coordinates of point P that is ⅓ of the way from A(4,-3) to B(1,3). Write your answer as an ordered pair (x, y).

2.   One vertex of an equilateral triangle is at the origin. The 2nd vertex lies on the graph of $y = x^2$. The 3rd vertex lies on the positive portion of the x-axis. What is the area of this triangle?

3.   Find the coordinates of the point Q on a circle whose equation is $(x-6)^2 + (y-5)^2 = 25$ such that Q is the nearest to the point (-2,11). Write your answer as an ordered pair (x, y).

ANSWERS

(1 pt.)   1.   P = (,   )

(2 pts.)   2.   _____

(3 pts.)   3.   Q = (,   )

Bartlett, Bromfield, Algonquin Regional

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   Find the area of a circle inscribed in a rhombus whose perimeter is 100 cm, and whose longer diagonal is 40 cm.

2.   The medians of a right triangle that are drawn from the vertices of the acute angles have lengths of $\sqrt{73}$ and $2\sqrt{13}$. Find the length of the hypotenuse.

3.   If $(x + 1)$ is a factor of $2x^2 + 5x + C$, find the value of $|\,\mathbf{5C}\,|$ **.**

4.   If $f(x) = x + 4$ and $f(g(x)) = -2x – 5$, find $g(f(x))$.

5.   How many 4-digit numbers can be formed using the digits 1,2,3,4,5,6, if the number is odd, between 3000 and 5000, and digits can be repeated?

6.   Find the equation of the line, with negative slope, which is tangent to the circle whose equation is: $x^2 + y^2 = 25$ at the point where $x = 3$. Write your answer in the form $Ax + By = C$ where A, B, and C are integers and A>0.

7.   Find the area of the circle passing through the points (0, 4), (4.4, 1.8), and (0, -7). Write your answer as a reduced improper fraction in the form $\dfrac{A\pi}{B}$.

8   The x-intercept of a line is twice the y-intercept and the line passes through the point (5, -2). Find **ALL** possible values for the x-intercept.

9.   Find the number of positive integers less than 1000 in which the sum of the digits of the number is 8. (e.g. "71", "521", "440")

 Tantasqua Regional, Tahanto, Westborough, Worcester Academy, Quaboag, Mass Academy

Round 1: Similarity and Pythagorean Theorem

 1.   **(1 pt.)** 9π

2.   **(2 pts.)**   **168/25**

3.   **(3 pts.)** $4 + 2\sqrt{2}$

Round 2: Algebra 1-open

1.   **(1 pt.)**   x - 1

2.   **(2 pts.)**   4

3.   **(3 pts.)**   5

Round 3: Functions

1.   **(1 pt.)** $\dfrac{(x + 1)}{2}$

2.   **(2 pts.)**   0, -¼

3.   **(3 pts.)** 3^{16}

Round 4: Combinatorics

1.   **(1 pt.)**   180

2.   **(2 pts.)**   2 and 6

3.   **(3 pts.)**   60

Round 5: Analytic Geometry

1.   **(1 pt.)**   (3, -1)

2.   **(2 pts.)** $3\sqrt{3}$

3.   **(3 pts.)**   (x,y) = (2,8)

TEAM ROUND (2 pts. Each)

1. 144π

2. 10

3. 15

4. $-2x - 17$

5. 216

6. $3x + 4y = 25$

7. $\dfrac{121\pi}{4}$

8. 1 and 0

9. 45

TEAM ROUND   **School: _____**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.   (2 points each)

1.   _____

2.   _____

3.   _____

4.   _____

5.   _____

6.   _____

7.   _____

8.   _____

9.   _____

_____

Total Points for Team Round: _____

February 4, 2004   **WOCOMAL Varsity Meet**

TEAM ROUND   **School: _____**

Team #:_____

Team Members:

1. _____

2. _____

3. _____

4. _____

5. _____